ICAB – failure criteria for beams (CM66)

The software ICAB Force/CM calculates the following criteria:
Sc tensile stress criterion (CM66, CB71)
Tc shear stress/(0.65 S0), (CM66)
Mc Mises criterion (or Tsai-Wu)
F_cm66 buckling criterion (CM66 ou CB71)
D_cm66 buckling and warping criterion (CM66)
V_cm66 I-web warping criterion (CM66)

Definition of allowable stress

The standards used to define allowable stresses in members are REGLES CM 66 (AFNOR P 22-701) and CB71 (AFNOR P21-701).

Stresses are computed from resultant stresses (axial Nx, shear Ty, Tz, torsion Tx, bending My, Mz) with using the following formulas:

$$\sigma_{Nx} = \frac{N_x}{A}, \quad \sigma_{fy} = \frac{M_y}{\left(\frac{I}{v}\right)_y}, \quad \sigma_{fz} = \frac{M_z}{\left(\frac{I}{v}\right)_z}$$

$$\tau_T = \frac{M_x}{\left(\frac{J}{r_0}\right)}, \quad \tau_y = \frac{T_y}{A_y}, \quad \tau_z = \frac{T_z}{A_z}$$

These stresses define the upper bounds of stresses σ_{xx}, τ_{xy} et τ_{xz}.

$$M_y = M_z = 0 \Rightarrow \sigma_{xx} = \sigma_{Nx}$$
$$N_x = 0, M_z = 0 \Rightarrow |\sigma_{xx}| \le |\sigma_{fy}|$$
$$N_x = 0, M_y = 0 \Rightarrow |\sigma_{xx}| \le |\sigma_{fz}|$$
$$T_y = T_z = 0 \Rightarrow \tau_{xy}^2 + \tau_{xz}^2 \le \tau_T^2$$
$$T_z = 0, M_x = 0 \Rightarrow |\tau_{xy}| \le |\tau_y|$$
$$T_y = 0, M_x = 0 \Rightarrow |\tau_{xz}| \le |\tau_z|$$

For a beam such as rectangular sections, the axial stress and shear are lesser than:

$$|\sigma_{xx}| = \sigma \le |\sigma_{Nx}| + |\sigma_{fy}| + |\sigma_{fz}|$$
$$\tau_{xy}^2 + \tau_{xz}^2 = \tau^2 \le \tau_1^2 + \tau_2^2$$
$$\tau_1 = |\tau_T| + \max(|\tau_y|, |\tau_z|)$$
$$\tau_2 = \min(|\tau_y|, |\tau_z|)$$

For rounded beam sections:

$$|\sigma_{xx}| = \sigma \le |\sigma_{Nx}| + \sqrt{\sigma_{fy}^2 + \sigma_{fz}^2}$$
$$\tau \le |\tau_T| + \sqrt{\tau_y^2 + \tau_z^2}$$

2.1 traction (**Sc**)

Safety criterion is:

$$\boldsymbol{\sigma/\sigma_e < 1}$$

Where σ is the maximum axial stress (CM66 1,312), i.e. σ is the sum $\sigma_{Nx}+\sigma_{fy}+\sigma_{fz}$.

2.2 Shear (**Tc**)

$$\boldsymbol{\tau/(0.65\ \sigma_e) < 1}$$

Where τ is the maximum shear stress (CM66 1,313), lesser than $\tau^2 = \tau^2_1+\tau^2_2$.

2.3 Von Mises criterion (**Mc**)

The Von Mises criterion (noted Mc) is used to check whether an isotropic material is yielding. For a symmetrical stress tensor, we have:

$$\frac{(\sigma_{xx}-\sigma_{yy})^2+(\sigma_{xx}-\sigma_{zz})^2+(\sigma_{yy}-\sigma_{zz})^2+6(\tau_{xy}^2+\tau_{xz}^2+\tau_{yz}^2)}{2\sigma_e^2} \le 1$$

$$\Leftrightarrow \frac{\sigma_{xx}^2+\sigma_{yy}^2+\sigma_{zz}^2-\sigma_{xx}\sigma_{yy}-\sigma_{yy}\sigma_{zz}-\sigma_{xx}\sigma_{zz}+3\left(\tau_{xy}^2+\tau_{xz}^2+\tau_{yz}^2\right)}{\sigma_e^2} \le 1$$

For a beam, we obtain:

$$\frac{\sigma_{xx}^2+3(\tau_{xy}^2+\tau_{xz}^2)}{\sigma_e^2} \le \frac{\sigma^2+3\,\tau^2}{\sigma_e^2} \le 1$$

For a pipe under pressure, i.e. with a membrane stress σ_{tt}, the Von Mises criterion is:

$$\frac{\sigma_{xx}^2+\sigma_{tt}^2-\sigma_{xx}\cdot\sigma_{tt}+3\,\tau^2}{\sigma_e^2} \le 1,\ldots avec\ldots\sigma_{xx}=\sigma_{Nx}\pm\sigma_f$$

with:

σ_{Nx} tensile stress related to axial force

σ_f stress related to bending moments

NB. The Von Mises criterion reported by ICAB is the square root of the preceding formula so that this criterion is proportional to applied load.

2.4 Buckling (**F_cm66**)

For a beam submitted to flexion and bending, the section 3,521 of CM66 (noted F_cm66) indicates that we must have:

$$\frac{k_1\sigma_{Nx}+k_{fy}\sigma_{fy}+k_{fz}\sigma_{fz}}{\sigma_e} \le 1$$

The coefficient k_1 corresponds to the buckling coefficient for exceptional checking (CM66 3,412).

$$k_1=\frac{\mu_1-1}{\mu_1-1.3},\ \mu_1=\frac{\sigma_k}{\sigma_{Nx}}$$

$$\sigma_k=\frac{\pi^2 E}{\lambda},\ \lambda=\max(\lambda_y,\ \lambda_z)$$

$$\left(I_{yz}\neq 0 \Rightarrow \lambda=\lambda_v\right)$$

The coefficient k_{fy} for the amplification of bending stresses is the most unfavorable corresponding to a constant bending moment of with a linear variation (CM66 3,513):

$$k_{fy} = \frac{\mu_y + 0.25}{\mu_y - 1.3}, \ \mu_y = \frac{\sigma_{ky}}{\sigma_{Nx}}$$

$$\sigma_{ky} = \frac{\pi^2 E}{\lambda_y}, \ \lambda_y = \frac{l_{ky}}{\sqrt{\frac{I_{yy}}{A}}}$$

Similar calculations are conducted for compression and bending M_z.

2.5 Warping (**D_cm66**)

The envelope formula CM66 3,731 (noted D_cm66) is used:

$$\frac{\sigma_{Nx} k + \sigma_{fy} k_{fy} k_{dy} + \sigma_{fz} k_{fz} k_{dz}}{\sigma_e} \le 1$$

The stress σ_{Nx} is taken into account if the beam is subject to compression.

The buckling coefficient k is (CM66 3,411):

$$k = (0.5 + 0.65 \frac{\sigma_e}{\sigma_k}) + \sqrt{\left(0.5 + 0.65 \frac{\sigma_e}{\sigma_k}\right)^2 - \frac{\sigma_e}{\sigma_k}}$$

$$\sigma_k = \frac{\pi^2 E}{\lambda^2}, \ \lambda = \max(\lambda_y, \ \lambda_z)$$

$$\left(I_{yz} \neq 0 \ \Rightarrow \ \lambda = \lambda_v\right)$$

The warping coefficient Kdy is (CM66 3,611) :

$$\sigma_{dy} \ge \sigma_e \Rightarrow k_{dy} = 1$$

$$\sigma_{dy} < \sigma_e \Rightarrow k_{dy} = \frac{k_0}{1 + \frac{\sigma_{dy}}{\sigma_e}(k_0 - 1)}$$

$$\sigma_{dy} = \frac{\pi^2 E}{5.2} \frac{I_{zz}}{I_{yy}} \frac{h_z^2}{l_{Dy}^2} (D - 1) BC$$

$$k_0 = \left(0.5 + 0.65 \frac{\sigma_e}{\sigma_{k0}}\right) + \sqrt{\left(0.5 + 0.65 \frac{\sigma_e}{\sigma_{k0}}\right)^2 - \frac{\sigma_e}{\sigma_{k0}}}$$

$$\sigma_{k0} = \frac{\pi^2 E}{\lambda_0^2}, \ \lambda_0 = \frac{l_{Dy}}{h_z} \sqrt{\frac{4}{BC} \frac{I_{yy}}{I_{zz}} \left(1 - \frac{\sigma_{Dy}}{\sigma_e}\right)}$$

The coefficients B, C, D are used to take into account the distribution of loads and dimensions of the section:

$$D = \sqrt{1 + \frac{4}{\pi^2}\frac{GJ}{E\,I_{zz}}\frac{l_k^2}{h_z^2}}$$

$$C = \sqrt{\frac{3}{1 + \frac{M_e}{M_w} + \left(\frac{M_e}{M_w}\right)^2 - 0.152\left(1 - \frac{M_e}{M_w}\right)^2}}$$

$$B = \sqrt{1 + \left(\frac{z_a}{h_z}\frac{8\beta C}{\pi^2 D}\right)^2} - \frac{z_a}{h_z}\frac{8\beta C}{\pi^2 D}$$

The calculation of C is truly valid for beams subjected to bending moments at the supporting points with flanges free to rotate around local beam axis (z) ; M_w and M_e are these two bending moments, M_w being the highest. The coefficient B used for the calculation is the smallest of B values calculated at z_a=0, h/2. The coefficient b=3 corresponds to a uniform load distributed on a beam free to rotate around its extremities around local axis (z) (CM66 3,643).

2.6 Buckling lengths

The calculations of failure criteria use 4 lengths:
l_{ky}, l_{kz}: buckling lengths in the two local planes XoZ and XoY of the beam
l_{Dy}, l_{Dz}: warping lengths in the two local planes XoZ and XoY of the beam

Hence, the length l_{ky} is used for the bucking in the XoZ plane that may occur with stress resultant Nx (compression) and My (moment around y axis).

In this note, these lengths are at least equal to the length of the finite element beam, i.e. distance between nodes. These lengths may be increased by specifications defined in physical properties of the beam:
LKY: minimum buckling length
LKZ: minimum buckling length
LDY: minimum warping length
LDY: minimum warping length

In all members used in the model, the buckling and warping lengths are at least equal to the length of the beam between nodes.
For horizontal bracing, the out-of-plane buckling length is the distance between legs.

NB. The program used for the present simulations computes the criteria F_cm66 (buckling) and D_cm66 (buckling and warping) for all sections, even for hollow sections, for which the warping failure may be not relevant.